Kinetic Modulational Instability of Upper-hybrid Turbulence

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It is shown that a plasma containing randomly distributed non-interacting upper-hybrid waves can become unstable against ion quasi-modes. The growth rate of the instability is presented.

In previous papers [1, 2], it has been shown that the upper-hybrid turbulence consisting of an ensemble of random-phased upper-hybrid waves can become modulationally unstable with respect to low-frequency ion-cyclotron, lower-hybrid [1], and adiabatic perturbations [2]. The latter is valid for the quasi-static régime.

In this letter, we consider the problem of modulation of upper-hybrid turbulence by low-frequency perturbations (Ω, q). A general dispersion relation which is valid for the quasi-static $(|\Omega/q\,v_{\mathrm{Ti}}| \ll 1)$, the inertial $(|\Omega/q\,v_{\mathrm{Ti}}| > 1)$, and the transitional $(|\Omega/q\,v_{\mathrm{Ti}}| \sim 1)$ regimes is obtained. Specificially, we shall be concerned with the last régime, namely the problem of coupling of upper-hybrid turbulence with ion quasi-modes. For the latter type of perturbations, the two fluid description fails and one should retain the Vlasov description.

In what follows, the high-frequency upper-hybrid waves shall be allowed to have a small component E_z along the external magnetic field $B_0\,\hat{z}$ so that they may couple with the ion quasi-modes. The dynamics of the upper-hybrid turbulence is governed by a wave kinetic equation [1, 3]. The change of the upper-hybrid turbulence distribution $\tilde{N}_k$ in the presence of ion quasi-mode perturbations $\tilde{n}_e$ is given by [1]

$$\tilde{N}_k = -\frac{\omega_k}{2}\,\frac{\tilde{n}_e}{n_0}\,q\cdot\frac{\partial N_k{}^0/\partial k}{\Omega - q\cdot v_g}, \qquad (1)$$

where $N_k{}^0 = \langle|E_k{}^0|^2\rangle/4\,\pi\,\omega_k$ is the unperturbed distribution, and

$$\omega_k^2 = \omega_{\mathrm{pe}}^2 + \omega_{\mathrm{ce}}^2 + 3\,k^2\,v_{\mathrm{Te}}^2\,\omega_{\mathrm{pe}}^2/(\omega_{\mathrm{pe}}^2 - 3\,\omega_{\mathrm{ce}}^2)$$

is the characteristic frequency of the turbulence. Here v_{Te}, ω_{pe}, and ω_{ce} are respectively the thermal velocity, plasma and gyrofrequencies of the electrons. The group velocity of the high-frequency waves is given by

$$v_g = \hat{x}\,[3\,k\,\lambda_e\,v_{\mathrm{Te}}\,\omega_{\mathrm{pe}}/\omega_k\,(1 - 3\,\omega_{\mathrm{ce}}^2/\omega_{\mathrm{pe}}^2)]\,, \qquad (2)$$

where $\lambda_e = v_{\mathrm{Te}}/\omega_{\mathrm{pe}}$ is the electron Debye length. We note that for $3\,\omega_{\mathrm{ce}}^2 \ll \omega_{\mathrm{pe}}^2$, the upper-hybrid modes have positive group dispersion, whereas for $\omega_{\mathrm{pe}}^2 \ll 3\,\omega_{\mathrm{ce}}^2$, the modes have negative group dispersion. In the following, we consider only the upper-hybrid waves with positive group dispersion, namely $3\,\omega_{\mathrm{ce}}^2 \ll \omega_{\mathrm{pe}}^2$.

The charge density perturbation ϱ_e in the presence of the modified distribution (1) is obtained from the electron Vlasov equation. We find

$$4\,\pi\,\varrho_e = -q^2\,\chi_e\,(\Phi + \Phi_{\mathrm{pe}}) \qquad (3)$$

where Φ is the ambipolar potential, and the ponderomotive potential Φ_{pe} is given by

$$\Phi_{\mathrm{pe}} = -\sum_k \pi\,e\,\omega_k\,\tilde{N}_k/m_e(\omega_k^2 - \omega_{\mathrm{ce}}^2)\,. \qquad (4)$$

In (3), $\chi_e = (q\,\lambda_e)^{-2}\,G'\,(\Omega/q\,v_{\mathrm{Te}})$ is the electron susceptibility, and G is the plasma dispersion function. The electrons are assumed to be highly magnetized.

From the linearized ion Vlasov equation, we obtain

$$4\,\pi\,\varrho_i = -q^2\,\chi_i\,\Phi\,, \qquad (5)$$

where $\chi_i = (q\,\lambda_i)^{-2}\,G'\,(\Omega/q\,v_{\mathrm{Ti}})$ is the ion susceptibility, and ions are assumed to be unmagnetized. The ponderomotive potential Φ_{pi} of the ions is smaller than that of the electrons by a factor m_e/m_i and is therefore neglected.

Combining (3) and (5), and using Poisson's equation $q^2\,\Phi = 4\,\pi\,(\varrho_i + \varrho_e)$, one gets

$$\Phi = -\chi_e\,\Phi_{\mathrm{pe}}/\varepsilon\,, \qquad (6)$$

where $\varepsilon = 1 + \chi_e + \chi_i$. Inserting (6) into (3), we get

$$\frac{\tilde{n}_e}{n_0} = \frac{q^2\,\chi_e\,(1+\chi_i)}{4\,\pi\,n_0\,e\,\varepsilon}\,\Phi_{\mathrm{pe}}\,. \qquad (7)$$

Combining (1) and (7), we obtain the dispersion relation

$$1 = \frac{L}{16\,\pi\,n_0}\,\frac{q^2\,\chi_e\,(1+\chi_i)}{m_e\,\varepsilon}\,\frac{(\omega_{\mathrm{pe}}^2 + \omega_{\mathrm{ce}}^2)}{\omega_{\mathrm{pe}}^2} \cdot \int \frac{q\cdot\partial N_k{}^0/\partial k}{(\Omega - q\cdot v_g)}\,\mathrm{d}k\,, \qquad (8)$$

where the summation over k has been replaced by an integration in the usual manner (i. e., $\sum_k \mathrm{d}k \rightarrow L/2\,\pi\int\mathrm{d}k$, where L is the size of the system). Equa-

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tion (8) is the most general dispersion relation describing the interaction of upper-hybrid turbulence with low-frequency perturbations.

For $|\Omega| \ll q\, v_{\mathrm{Te}}$, we have $\chi_{\mathrm{e}} = (q\,\lambda_{\mathrm{e}})^{-2}$. We now let

$$N_k{}^0 \approx (2\,\pi)^{1/2}\,(W/4\,\pi\,\omega_{\mathrm{pe}})\,(k_{\mathrm{t}}\,L)^{-1} \cdot \exp\left[-(k-k_0)^2/2\,k_{\mathrm{t}}{}^2\right]$$

in (8) and assume that the spectrum of upper-hybrid turbulence is sufficiently peaked around k_0 [i. e., $|\Omega - q_x\,u_0| > 3\,q_x\,\lambda_{\mathrm{e}}\,k_t\,v_{\mathrm{Te}}\,\omega_{\mathrm{pe}}/(\omega_{\mathrm{pe}}^2 + \omega_{\mathrm{ce}}^2)^{1/2}$, where $u_0 = v_{\mathrm{g}}(k = k_0)$], we then obtain from (8)

$$\Omega - q_x\,u_0 = \pm i\,(3/8\,\pi)^{1/2}\,q_x\,\lambda_{\mathrm{e}}\,\omega_{\mathrm{pe}} \cdot \left(\frac{W}{n_0\,T_{\mathrm{e}}}\right)^{1/2} \left(\frac{1+\chi_{\mathrm{i}}}{\varepsilon}\right)^{1/2} A\,, \qquad (9)$$

where k_t is the spread, k_0 is the mean wave vector, W is the total energy of the turbulence spectrum, and $4\,A^2 = \omega_{\mathrm{H}}\,\omega_{\mathrm{pe}}/(\omega_{\mathrm{pe}}^2 - 3\,\omega_{\mathrm{ce}}^2)$ with $\omega_{\mathrm{H}}^2 = \omega_{\mathrm{pe}}^2 + \omega_{\mathrm{ce}}^2$.

Letting $\Omega = q_x\,u_0 + i\,\gamma$, we obtain the growth rate

$$\gamma = (3/8\,\pi)^{1/2}\,q_x\,\lambda_{\mathrm{e}}\,(W/n_0\,T_{\mathrm{e}})^{1/2}\;\omega_{\mathrm{pe}}\,A\,R_{\mathrm{e}} \cdot \left[\left(\frac{1+\chi_{\mathrm{i}}}{\varepsilon}\right)\right]^{1/2}_{\Omega = q_x u_0}, \qquad (10)$$

where the frequency shift caused by the turbulence is neglected. When the argument of χ_{i} is near unity, that is $3\,k_0\,\lambda_{\mathrm{e}}\,v_{\mathrm{Te}} \approx v_{\mathrm{Ti}}$, which can occur for $k_0\,\lambda_{\mathrm{e}} \ll 1$, we find for $T_{\mathrm{e}} \approx T_{\mathrm{i}}$,

$$\gamma \approx 0.9\,(3/8\,\pi)^{1/2}\,q_x\,\lambda_{\mathrm{e}}\,(W/n_0\,T_{\mathrm{e}})^{1/2}\,\omega_{\mathrm{pe}}\,A\,. \qquad (11)$$

In conclusion, we have shown that a spectrum of upper-hybrid mode turbulence is unstable when the group velocity of the turbulence mode is approximately equal to the ion thermal speed.

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[1] P. K. Shukla and M. Y. Yu, Phys. Lett. **57 A,** 57 [1976].
[2] K. H. Spatschek, Phys. Lett. **57 A,** 333 [1976].
[3] B. B. Kadomtsev, Plasma Turbulence, Academic Press, London 1965, p. 34.